Lab Activity: Continuous Random Variable

In this lab activity, you will compute uniform and exponential probabilities using Statcato.

Student Learning Outcomes

By the end of this chapter, you should be able to:

- Compute uniform and exponential probability density, cumulative probability, and inverse cumulative probability using Statcato
- Recognize the uniform and exponential probability distribution and apply it appropriately

Preliminary

Read Chapter 5 Continuous Random Variables in:

Illowsky, Barbara, and Susan Dean. Collaborative Statistics. Connexions. <http://cnx.org/content/col10522/latest>.

Make sure you understand the following **key terms** (**LR:Key Terms**):

continuous random variable, uniform probability distribution, exponential probability distribution, probability density function, cumulative distribution function

Using Uniform Probability Distributions

Suppose the number of hours statistics students at your school study per day follow a uniform distribution between 0 and 8. Let X be the number of hours a statistics student studies per day. We will answer the following questions (by hand and using Statcato).

What is the probability density function?

X follows a uniform distribution where the lowest value is 0 and the highest value is 8. Thus, $X \sim U(0, 8)$. Therefore, $f(X) = \frac{1}{8-0} = \frac{1}{8}$ for $0 \le X \le 8$.

What is the mean and standard deviation of the distribution?

The mean is $\mu = (0+8)/2 = 4$.

The standard deviation is $\sigma = \sqrt{\frac{(8-0)^2}{12}} = \sqrt{\frac{64}{12}} \approx 2.309$.

What is the probability that a randomly selected statistics student studies no more than four hours per week?

$P(X \le 4) = 1/8\ (4-0) = 1/2$. You can also calculate the cumulative probability using Statcato.

Calculating Uniform Cumulative Probability

Go to `Calculate > Probability Distributions > Uniform` (or select from Dialog History).

- **Distribution Parameters**
  - Lower bound: 0
  - Upper bound: 8
- **Compute**: Cumulative probability
- **Input(s)**: Select Constant. Enter 4 in the provided text box.
- Click **Compute**.

The cumulative probability $P(X \leq 4)$ is shown in the Log.

Find the 70th percentile for a statistics student's studying time per day.

$P(X < k) = 0.70$. $(1/8)k = 0.70$. Therefore, $k = 5.6$. The 70th percentile is 5.6 hours. Using Statcato, you can find the inverse cumulative probability (i.e. find k given $P(X < k)$).

Calculating Uniform Inverse Cumulative Probability

Go to `Calculate > Probability Distributions > Uniform` (or select from Dialog History).

- **Distribution Parameters**
  - Lower bound: 0
  - Upper bound: 8
- **Compute**: Inverse cumulative probability
- **Input(s)**: Select Constant. Enter 0.7 in the provided text box.
- Click **Compute**.

The inverse cumulative probability is shown in the Log.

Using Exponential Probability Distributions

Suppose that the length of time that a laptop battery lasts is exponentially distributed, and the average life of the battery is 7 years. Let X be the amount of time in years a laptop battery lasts. We will answer the following questions (by hand and using Statcato).

What is the probability density function?

X follows an exponential distribution where the decay parameter is $1/7 \approx 0.142857$. Thus, $X \sim Exp(1/7)$. Therefore, $f(X) = \frac{1}{7} e^{-\frac{X}{7}}$, $X \geq 0$.

What is the mean and standard deviation of the distribution?

The mean and standard deviation are the same: $\mu = \sigma = 7$.

What is the probability that a randomly selected battery lasts no more than 5 years?

$P(X \le 5) = 1 - e^{-\frac{5}{7}} \approx 0.5105$. You can also calculate the cumulative probability using Statcato.

Calculating Exponential Cumulative Probability

Go to `Calculate > Probability Distributions > Exponential` (or select from Dialog History).

- **Distribution Parameters**
  - Rate: 0.142857
- **Compute**: Cumulative probability
- **Input(s)**: Select Constant. Enter 5 in the provided text box.
- Click **Compute**.

The cumulative probability $P(X \le 5)$ is shown in the Log.

Find the 80th percentile for the battery life.

$k = -\ln(1 - 0.8) / (1/7) \approx 11.266$. So the 80th percentile is 11.266 years. Using Statcato, you can find the inverse cumulative probability (i.e. find k given $P(X < k)$).

Calculating Exponential Inverse Cumulative Probability

Go to `Calculate > Probability Distributions > Exponential` (or select from Dialog History).

- **Distribution Parameters**
  - Rate: 0.142857
- **Compute**: Inverse cumulative probability
- **Input(s)**: Select Constant. Enter 0.8 in the provided text box.
- Click **Compute**.

The inverse cumulative probability is shown in the Log.

Problems

Answer the following questions in **LR: Problems**. Perform calculations of probabilities by hand and using Statcato. Show your work as well as computer-generated results from Statcato.

1. Suppose the hourly rate of a student job is uniformly distributed between \$7 and \$30.
   a. What is the probability density function?
   b. What is the mean and standard deviation of the distribution?
   c. What is the probability that a randomly chosen student job pays more than \$10?

d. Find the 95^{th} percentile of the hourly rate.

e. Find the probability that a student job pays more than $25 given that its hourly rate is more than $10.

f. Find the probability that a student job pays between $10 and $25.

2. Uranium-232 is a radioactive isotope of Uranium with a half-life of about 68.9 years. Here we model the decay of Uranium-232 using an exponential decay rate of 0.01006. Suppose you start with 1 gram of Uranium-232.

   a. What is the probability density function?

   b. What is the mean and standard deviation of the distribution?

   c. How much of Uranium-232 (in grams) last less than 30 years?

   d. How much of Uranium-232 (in grams) last more than 70 years?

   e. How much of Uranium-232 (in grams) last between 30 and 70 years?

   f. Within how many years will 80% of Uranium-232 decay?